1 The mean free path in a complex medium

We suppose a complex medium composed of n elements with mass number and atomic number (A_i, Z_i) $(i = 1, n)$. Let the relative number of them be N_i. (Say, for BGO, $A_i \approx 209, 73, 16, Z_i = 83, 32, 8, N_i = 4, 3, 12$. For Air, $A_i = 14, 16, 40, Z_i = 7, 8, 18, N_i = 1.56, 0.42, .01$; note that the volume abundance of N_2,O_2, Ar is 78,21,1 %, respectively) Also we use the normalized number abundance of elements;

$$r_i = \frac{N_i}{\sum N_i} \tag{1}$$

We disregard the possible coherence of molecules etc. Therefore we can treat each element as an independent target. Then the cross-section of the target is

$$\sigma = \sum r_i \sigma_i \tag{2}$$

where σ_i is the cross-section of the i-th element. We call "one unit of the target" (molecule or atom or aggregate such as 1.56 N +0.42 O + 0.01Ar for Air) as "pseudo molecule". Then, the atomic weight of the psuedo molecule is

$$<\!A\!> \equiv \sum r_i A_i \tag{3}$$

One "pseudo molecule" has the weight of $<\!A\!>/N_A$ g, where N_A is the Avogadro number. Therefore the the number of pseudo molecules in the unit volume is

$$\frac{\rho N_A}{<\!A\!>} \tag{4}$$

where ρ is the density of the target. The mean free path for the collision, λ, is obtained from the relation

$$\lambda \frac{\rho N_A}{<\!A\!>} \sigma = 1 \tag{5}$$

i.e.,

$$\frac{1}{\lambda} = \rho N_A \frac{\sigma}{<\!A\!>} \tag{6}$$

The density contribution from the i-th element is

$$\rho \frac{r_i A_i}{<\!A\!>} \tag{7}$$

so that the mean free path for collision with the i -th element is

$$\frac{1}{\lambda_i} = \rho N_A \frac{r_i A_i}{<\!A\!>} \frac{\sigma_i}{A_i} = \rho N_A \frac{r_i \sigma_i}{<\!A\!>} \tag{8}$$

Hence

$$\frac{1}{\lambda} = \sum \frac{1}{\lambda_i} \tag{9}$$

We note that λ_i can also be represented by using the weight fraction, w_i, of the i-th element as

$$w_i = r_i A_i / <\!A\!> \tag{10}$$

$$\frac{1}{\lambda_i} = \rho N_A w_i \frac{\sigma_i}{A_i} \tag{11}$$

In all expressions for the mean free path, if we drop ρ, we get the mean free path in g/cm^2, provided that length scale is expressed in cm.

Once collision path is sampled using λ, we can fix the target element with the relative ratio of $1/\lambda_i$, i.e, $i-$th element is taken with the probabilty of $\frac{\lambda}{\lambda_i} = \frac{r_i \sigma_i}{\sigma}$.

2 Program interface

For a given medium, we prepare

r_i	media.No(i)
A_i	media.elem(i).A
Z_i	media.elem(i).Z
n	media.noOfElem
$w_i = \frac{r_i A_i}{<A>}$	media.w(i)
r_i	media.rn(i)
$\sum w_i A_i$	meida.Aeff
$<A>= \sum r_i A_i$	media.A
$<Z>= \sum r_i Z_i$	media.Z
$<Z^2>= \sum r_i Z_i^2$	media.Z2
σ_i	media.elem(i).sigma
$\sigma = \sum r_i \sigma_i$	meida.sigma
$\frac{\rho N_A}{<A>}$	media.ndensity
λ	media.mfp

Since σ_i changes with energy and projectile particles, we have to calculate them every time we need the mean free path. The unit of σ is in mb and $\rho N_A/ <A>$ is cm^{-3}.

3 Program structure

- Suppose the particle information is given by a /ptcl/ redord, 'p', in which we find particle type, energy, mass, charge etc.
- Suppose the target information is given by $n, A_i, Z_i, r_i, \sum r_i A_i, \rho N_A/ <A>$
- From 'p' and the target info. we compute σ_i and σ
- Then, λ is computed.

Since it is time consuming to compute σ_i for a large number of elements every time, we may tabulate σ as a function of σ_{hp} (h = a simple hadron) cross-section which is varied from 10 mb to 100 mb with a step of 2 mb. For other cases, we may compute σ_i every time.

The procedure for making talbe is

- For σ_{hp}=10 to 100 step 2mb, do the following
- For every target A_i, obtain σ_i by converting σ_{hp}
- Get $\sigma = \sum r_i \sigma_i$

When using the table, do the following

- If a projectile particle is not a simple hadron but a nucleus, then don't use the table.
- For a simple hadron, compute the cross-section for the proton target (σ_{hp}).
- If σ_{hp} dosen't fall in 10 to 100 mb, don't use the table.
- Make the linear interpolation of the table using σ_{hp}.