1 How to convert simulation data into flux values

Let the solar modulated primary energy spectrum of species, i (=p, He, CNO,etc), be I_{0i} and the rigidity cut function $R_i(E,\theta,\varphi)$, where θ is the zenith angle of the primary and φ the azimuthal angle, E the energy. $R_i(E,\theta,\varphi)$ takes 0 or 1. We assume that every particle is always obseved with an agnle perpendicular to the particle direction in Monte Carlo calculations.

The number of primaries, ΔN, directed to pass through a horizontal area, S, in a time interval, T, with a solid angle range, $\Delta\Omega$ and an energy bin, ΔE is

$$\Delta N = ST\sum_i I_{0i}R_i(E,\theta,\varphi)\cos\theta\Delta\Omega\Delta E \tag{1}$$

In other words, if a simulation produces, N, primaries within a solid angle range $\Delta\Omega$ and with a threshold energy E_i^{th},

$$N = ST\sum_i \int_{\Delta\Omega} d\Omega \int_{E_i^{th}} dE\cos\theta I_{0i}R_i(E,\theta,\varphi) \tag{2}$$

From this Eq.**??**, we can obtain ST.

In this simulation, if we observe ΔN particles in $\Delta\Omega$ and ΔE at some depth, the flux, $F(\theta,\varphi)$, of the particles should be

$$\Delta N = STF(\theta,\varphi)\cos\theta\Delta\Omega\Delta E \tag{3}$$

Normally observation disregards the azimuthal angle and we may always integrate the flux over the azimuthal angle and get the flux as the average over the azimuthal angle. In this case, ΔN is the number observed in $\Delta\cos\theta$ and ΔE disregarding azimuthal angle.

$$F(\theta) = \frac{1}{2\pi}\int_0^{2\pi} F(\theta,\varphi)d\varphi \tag{4}$$

$$= \frac{1}{ST\cos\theta}\frac{\Delta N}{\Delta E 2\pi\Delta\cos\theta} \tag{5}$$

For a finite range of $\cos\theta$ ($\cos\theta_1$ to $\cos\theta_2$), we may define the average flux in this range $\bar{F}(E)$ as

$$\bar{F}_\theta(E)\int_{\cos\theta_1}^{\cos\theta_2}\cos\theta d\cos\theta = \int_{\cos\theta_1}^{\cos\theta_2} F(\theta)\cos\theta d\cos\theta \tag{6}$$

$$\bar{F}_\theta(E) = \frac{1}{2\pi ST}\frac{1}{\frac{1}{2}(\cos^2\theta_2 - \cos^2\theta_1)}\frac{\Delta N}{\Delta E} \tag{7}$$

Here ΔN means all the particles falling within the given cosine range in the ΔE bin.

The angular distribution of observed particles should be

$$F(\theta)2\pi ST\cos\theta = \frac{\Delta N}{\Delta E\Delta\theta} \tag{8}$$

Therefore, if the observed angular distribution is approximated by $Af(\cos\theta)$ (with $f(1) = 1$), we obtain

$$F(\theta) = \frac{Af(\cos\theta)}{2\pi ST\cos\theta} \tag{9}$$

or the vertical flux as

$$F(0) = \frac{A}{2\pi ST} \tag{10}$$

In some case, we observe particles above some energy, E. If the nubmer is, $N_\theta(> E)$, the corresponing integral flux is,

$$\bar{F}_\theta(> E) = \frac{1}{2\pi ST}\frac{1}{\frac{1}{2}(\cos^2\theta_2 - \cos^2\theta_1)}N_\theta(> E) \tag{11}$$

If we integrate over the angle,

$$\bar{F}(> E)\pi(\cos^2\theta_2 - \cos^2\theta_1) = \frac{N(> E)}{ST} \tag{12}$$

Similarily, the angular distribution of the integral flux is

$$F(\theta, > E)2\pi ST\cos\theta = \frac{\Delta N(> E)}{\Delta\theta} \tag{13}$$

Assume this is approximated by $Bg(\cos\theta)$, then

$$F(\theta, > E) = \frac{Bg(\cos\theta)}{2\pi ST\cos\theta} \tag{14}$$

($g(0) = 1$) and the vertial integral flux

$$F(0, > E) = \frac{B}{2\pi ST} \tag{15}$$

2 Average primary spectrum when a cutoff exists

Suppose zenith angle and azimuthal angle regions are given. Then, we want to see the average primary spectrum over that region. If there is no rigidity cutoff, the spectrum should completely be the same as the vertical one. However, the cutoff is a function of θ, ϕ, E_i,