How to convert simulation data into flux values

Contents

1 Introduction

Let the solar modulated primary energy spectrum of species, i (=p, He, CNO,etc), be I_{0i} (assumed to be completely isotropic) and the rigidity cut function $R_i(E,\theta,\varphi)$, where θ is the zenith angle of the primary and φ the azimuthal angle, E the energy. $R_i(E,\theta,\varphi)$ takes 0 to 1 (probability of entering into the atmosphere).

The number of primaries, ΔN, directed to pass through a detector in a time interval, T, with a small solid angle range, $\Delta\Omega$, and an energy bin, ΔE, is

$$\Delta N = T\sum_i I_{0i}R_i(E,\theta,\varphi)S(\theta,\varphi)\Delta\Omega\Delta E \tag{1}$$

where $S(\theta,\varphi)$ is the **cross-section area** of a detector (i.e, surface area projected to a plane perpendicular to the primary direction). We are implicitly supposing that the primary with (θ,φ) would produce secondary particles falling on the detector with the same angle. We will return to this problem shortly.

Typically we can raise four cases:

$$S(\theta,\varphi) = \begin{cases} S_0\cos\theta & \text{:a horizontal flat surface} \\ S_0 & \text{: a whole sphere} \\ S_0\frac{1+\cos\theta}{2} & \text{: an upper hemisphere} \\ S_0\frac{\cos\theta+\frac{2h}{\pi r}\sin\theta}{\sqrt{1+(\frac{2h}{\pi r})^2}} & \text{: a whole cylinder} \end{cases} \tag{2}$$

where S_0 denotes the maximum cross-section area which is seen at $\theta = 0$ for a flat surface and sphere, and at $\tan\theta = 2hr/\pi r^2$ for a cylinder, where r is the cylinder radius, h the height. The cross-section area of the cylinder is simply $\pi r^2\cos\theta + 2rh\sin\theta$ and the maximum area is $S_0 = \pi r^2\sqrt{1+(\frac{2hr}{\pi r^2})^2}$. The discussion below will be valid for other cases.

Generally, Cosmos samples the primaries from the spectrum of $I_{0i}R_idE_id\Omega$ and dose not take into account $S(\theta,\varphi)$. If a simulation produces, N, primaries within a finite solid angle range $\Delta\Omega$ and above a threshold energy E_i^{th}, we calculate the time period, T, corresponding to that simulation by

$$N = S_0T\sum_i\int_{\Delta\Omega}d\Omega\int_{E_i^{th}}dEI_{0i}R_i(E,\theta,\varphi) \tag{3}$$

We cannot, however, put all of these N primaries in the detector; we must discard some of them only by accepting each particle with the probability of S/S_0. The total number of primaries accepted will be

$$N' = N\frac{\sum_i \int_{\Delta\Omega} d\Omega \int_{E_i^{th}} dES(\theta,\varphi)I_{0i}R_i(E,\theta,\varphi)}{S_0\sum_i \int_{\Delta\Omega} d\Omega \int_{E_i^{th}} dEI_{0i}R_i(E,\theta,\varphi)} \tag{4}$$

The program, getST.f, in *Cosmos*/Util/Flux/ is to compute

$$\sum_i \int_{\Delta\Omega} d\Omega \int_{E_i^{th}} dE\frac{S(\theta,\varphi)}{S_0}I_{0i}R_i(E,\theta,\varphi) \tag{5}$$

for the three cases and also to give the ratio in Eq.4

2 Vertical flux

Let us suppose we have generated a number of events by Cosmos and observed particles at a given depth. We want to estimate the flux of particles of some type at that depth. Suppose the flux of the particles in $\Delta\Omega$ and ΔE at that depth (before entering into a detector) is $F(\theta,\varphi)$. (That is, the number of particles passing through the unit area perpendicular to the particle direction in the unit time.) Then, if the simulation observes Δn particles in ΔE and $\Delta\Omega$ bins, we have

$$\Delta n = S_0TF(\theta,\varphi)\Delta\Omega\Delta E \tag{6}$$

This relation implicitly assumes that

A) the primary with (θ,φ) generates particles with the same angle. Or

B) if the particles are scattered away from $\Delta\Omega$ bin, there should be the same amount of particles entering into this bin from other angles. (The same is true in Sec.3). This will hold if the isotropy of the particles within the scattering angle is good. Generally, this is not the case especially at low energies and we have difficulty in deriving the correct flux by M.C.

Note that, if we use prescription B), and further put observed particles to a detector, we should assign the same angle to all the observed particles as the primary angle, although the observed particles have various scattered angles.

2.1 How to derive

1. The vertical flux is defined as $F(0,\varphi)$ where φ is arbitrary. To derive the vertical flux, one may perform a simulation with vertically incident primaries in one dimensional mode. This assumes A) above. Since we make $\Delta\Omega \to 0$ as $\theta \to 0$, we have to put $T \to \infty$ to get a finite number of events. So we may take that the product of $S_0T\Delta\Omega$ remains finite. From Eq. 3 , we have

$$S_0T\Delta\Omega = N/\sum_i \int_{E_i^{th}} dEI_{0i}R_i(E,\theta,\varphi) \tag{7}$$

The program, getST2.f, in *Cosmos*/Util/Flux, computes the sum in the right hand side of the above equation for a given (θ,φ). Using this $S_0T\Delta\Omega$ for $\theta = 0$, we obtain the vertical flux as

$$F(0) = \frac{\Delta n}{S_0T\Delta\Omega\Delta E} \tag{8}$$

2. Usually particles reach a given depth along scattered paths so that the one dimensional approximation underestimates the path length; this would lead to over estimation or underestimation of the flux depending on the development shape of the particles and the depth. (In the case of air shower development, the number of particles at the shower maximum by 3D calculation is lower by 30 % than the 1D case.). So assuming B) above, we may perform a 3D simulation fixing the primary direction vertical. We take all particles that reach the depth to compute Δn. This would probably give a better result than the one dimensional case.

Although the above argument is correct, due to the organization of the cutoff table, we may get different results depending on the choice of the azimuthal angle. Therefore **it is recommended to use getST3.f to get the vertical flux** described later or the methods in the next section.

2.2 Cutoff-free methods for the vertical flux derivation

1. Simulation should be performed with the vertical primary. If we draw the sampled primary spectrum in the form of dN/dE (*Cosmos*/Util/testSampPrim.f), and compare it with the absolute primary flux, dI/dE at $E >$cut off, we can obtain the conversion factor, A, so that $AdN/dE = dI/dE$. This conversion factor can be used to convert an observed dn/dE to the absolute scale.

2. If the total number of primaries is known in which the primaries discarded due to the cut-off are included, we get time, T, corresponding to that total number. $S_0\Omega T$, can be obtained analogous to the previous method.

2.3 Correcting the vertical flux

The vertical flux derived by the previously mentioned methods are very accurate above $\gtrsim$ 2 GeV. However, it may overestimate the flux at lower energies since the particles of such energies are deflected largely; here we are implicitly assuming primaries of large zenith angles are attenuated with the same rate for the vertical ones. However, this is not the case so a rough correction can be made by putting the attenuation factor $\exp((1 - 1/\cos\theta)Z/\Lambda)$ to the particles of zenith angle θ where Z is the vertical depth and Λ the effective attenuation length.

$\Lambda \sim 120$ g/cm^2 is too small at low energies and we may employ $\Lambda = 200 \sim 250$ g/cm^2 which gives more or less good result.

2.4 True 3D simulation

The true 3D simulation can be performed by using ObsPlane=3 and giving proper values to Azimuth and CosZenith (for definition of Azimuth and CosZenith, see the Cosmos manual). For particles other than neutrinos, we may limit the solid angle above the given observation point. Normally, the opening angle (at the earth center) could be as small as few degrees and the zenith angle region of the primaries $> 45°$: For example typical assignment would be

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Azimuth = (0.,0.6)
CosZenith = (0.7, 1.0)
HeightOfInj = 65.d3
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The vertical flux could be obtained by taking particles very close to the vertical and the detected point not far from the given observation point. The absolute flux can be obtained by normalizing the flux at > 2 GeV to the one obtained by the previously mentioned method.

However, if we limit the angle near to the vertical, statistics would not be enough. Therefore we may take particles with some finite angles. If the angular distribution is independent of the energy, we may take all the particles disregarding the angle. Unfortunately, this is not the case.

Let the angular distribution near the vertical be

$$V_i \cos^{n_i}\theta d\cos\theta \tag{9}$$

for an energy bin labeled by i. Upon integration from $\cos\theta = x_i$ to 1,

$$I_i = V_i \frac{1 - x_i^{n_i+1}}{n_i + 1} \tag{10}$$

If we use the same $x_i = 1 - \epsilon$ for all i and require ϵ be small,

$$\frac{I_i}{I_j} = \frac{V_i}{V_j} \cdot \frac{n_j + 1}{n_i + 1} \cdot \frac{1 - (1-\epsilon)^{n_i+1}}{1 - (1-\epsilon)^{n_j+1}} \tag{11}$$

$$\approx \frac{V_i}{V_j}(1 + \frac{n_j - n_i}{2}\epsilon) \tag{12}$$

If we want the ratio I_i/I_j to reflect the vertical flux ratio V_i/V_j within a few percent $(=\eta)$, we have to limit

$$\frac{|n_i - n_j|}{2}\epsilon \leq \eta/100 \tag{13}$$

That is the maximum zenith angle we can take is

$$\theta \approx \sqrt{\frac{\eta}{25\Delta n}} \tag{14}$$

where Δn is the maximum difference between n_i and n_j. Taking $\Delta n = 3$ and $\eta = 3$, we get $\theta = 1/5 = 11.5°$.

If the statistics is still not enough, we may take larger x_i and adjust it so that the ratio I_i/I_j is V_i/V_j. For the reference, we take the $i-$th energy bin. Then x_j should be

$$\cos\theta_j^{n_j+1} = x_j^{n_j+1} = 1 - \frac{n_j+1}{n_i+1}(1 - x_i^{n_i+1}) \tag{15}$$

If we take $x_i = 0.8$, $n_i = 6$ and $n_j = 3$, we get $x_j = 0.86$.

3 Trigger rate

Suppose that we have generated N primaries as explained in Sec.1. With these primaries, if N_{trig} events are triggered in a M.C simulation of detector response, the trigger rate would be

$$H_{trig} = N_{trig}/T \tag{16}$$

As noted previously, in the calculation of N_{trig}, we should discard some incident particles with the acceptance probability of S/S_0. Also, as noted in Sec.2, we should use the same angle for the secondary particles as the primary.

4 Flux from a given direction

Like the vertical flux, we can derive the flux from a given (θ, φ) direction, $F(\theta, \varphi)$. We may use the primaries from the given direction. This will be the case for high energy muon observation such as from the horizontal direction.

5 Simulation with various incident angles

Suppose we made a simulation with primaries of various incident directions as explained in Sec.1. The starting equation is the same as Eq. 6, i.e,

$$\Delta n = S_0 T F(\theta, \varphi)\Delta\Omega\Delta E \tag{17}$$

5.1 Flux from a given zenith angle

Normally observation disregards the azimuthal angle and we may always integrate the flux over the azimuthal angle. In this case, Δn is the number observed in $\Delta\cos\theta$ and ΔE bins disregarding azimuthal angle. However, it is better to get the flux per solid angle, so we may use the average flux over the azimuthal angle,

$$F(\theta) = \frac{1}{2\pi}\int_0^{2\pi} F(\theta, \varphi)d\varphi \tag{18}$$

$$= \frac{1}{S_0 T}\frac{\Delta n}{\Delta E 2\pi \Delta\cos\theta} \tag{19}$$

If we fix the primary zenith angle to θ while sampling φ from $(0, 2\pi)$, we may assume a finite $S_0 T 2\pi \Delta \cos\theta$ as in the case of the vertical flux; $S_0 T 2\pi \Delta \cos\theta$ can be computed by using Eq.3.

$$S_0 T 2\pi \Delta \cos\theta = N / \sum_i \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_{E_i^{th}} dE I_{0i} R_i(E, \theta, \varphi) \tag{20}$$

The summation in the right hand side is computed by the program, getST3.f. Naturally, upon making $\theta \to 0$, we get the vertical flux which is the same as derived previously.

5.2 Average flux in a zenith angle region

For a finite range of $\cos\theta$ ($\cos\theta_1$ to $\cos\theta_2$), we may define the average flux in this range $F(\bar{\theta})$ as

$$F(\bar{\theta}) \int_{\cos\theta_1}^{\cos\theta_2} d\cos\theta = \int_{\cos\theta_1}^{\cos\theta_2} F(\theta) d\cos\theta \tag{21}$$

or

$$F(\bar{\theta}) = \frac{1}{S_0 T} \frac{1}{\cos\theta_2 - \cos\theta_1} \frac{\Delta n}{\Delta E} \tag{22}$$

Here Δn means all the particles falling within the given cosine range in the ΔE bin.

6 The zenith angle distribution

The zenith angle distribution of the particles should be

$$F(\theta) 2\pi S_0 T = \frac{\Delta n}{\Delta E \Delta \cos\theta} \tag{23}$$

Therefore, if the angular distribution is approximated by $A(E) f(\cos\theta)$ (with $f(1) = 1$), we obtain

$$F(\theta) = \frac{A(E) f(\cos\theta)}{2\pi S_0 T} \tag{24}$$

or the vertical flux as

$$F(0) = \frac{A(E)}{2\pi S_0 T} \tag{25}$$

In some case, we observe particles above some energy, E. Then, the corresponding integral flux is

$$F(\bar{\theta}, > E) = \frac{1}{2\pi S_0 T} \frac{1}{\cos\theta_2 - \cos\theta_1} n(> E) \tag{26}$$

where $n(> E)$ is the particles with energy $> E$ in the given cosine region.

Similarily, the angular distribution of the integral flux is

$$F(\theta, > E) 2\pi S_0 T = \frac{\Delta n(> E)}{\Delta \cos\theta} \tag{27}$$

Assume this is approximated by $B(E) g(\cos\theta)$ ($g(1) = 1$), then

$$F(\theta, > E) = \frac{B(E) g(\cos\theta)}{2\pi S_0 T} \tag{28}$$

and the vertical integral flux

$$F(0, > E) = \frac{B(E)}{2\pi S_0 T} \tag{29}$$

7 Average primary spectrum when a cutoff exists

Suppose zenith angle and azimuthal angle regions are given. Then, we want to see the average primary spectrum over that region. If there is no rigidity cutoff, the spectrum should completely be the same as the vertical one.

The definition is

$$\bar{I}_{0i}(E) = \int_{\Delta\Omega} d\Omega I_{i0} R_i(E,\theta,\varphi) / \int_{\Delta\Omega} \tag{30}$$

and can be obtained by using the program, showSpec.f in *Cosmos*/Util/Flux.